AIR-1 Notes

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Fluid Mechanics

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FLUID MECHANICS

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Fluid Mechanics

1. Properties of Fluid

2. Fluid Statics
   - Fluid Pressure and its measurement
   - Hydrostatic forces on plane and curved surface
   - Liquid in relative equilibrium
   - Buoyancy and Floatation

3. Fluid kinematics
4. Fluid Dynamics
   - energy equations
   - momentum.
5. Flow measurement
6. Laminar Flow
7. Turbulent Flow
8. Boundary layer theory
9. Drag and lift
10. Model Analysis
11. Pipe flow.

Introduction

$\tau = G\gamma \rightarrow$ solids

$\tau \propto \dfrac{d\gamma}{dt} \Rightarrow \gamma = \tau t + c \rightarrow$ fluid.

1. Substance in liquid or gaseous phase is referred to as fluid. They are capable of deforming continuously under the influence of shear stress, however small the shear stress might be.

2. In solids $\boxed{\tau \propto \gamma}$ but in fluids $\boxed{\tau \propto \dfrac{d\gamma}{dt}}$

→ Study of fluid at rest is called fluid statics.

→ Study of fluid in motion, when forces responsible for motion are not the point of concern then the study is called fluid kinematics.

→ Study of fluid in motion when forces responsible for motion are the point of concern is called fluid dynamics.

⇒ Continuum Approach

→ We, in fluid mechanics, generally assume that fluid is a continuous homogenous mass with no holes. Thus, at every point in the flow space, we can define the flow parameters like velocity, acceleration etc.

→ This assumption is called continuum approach.

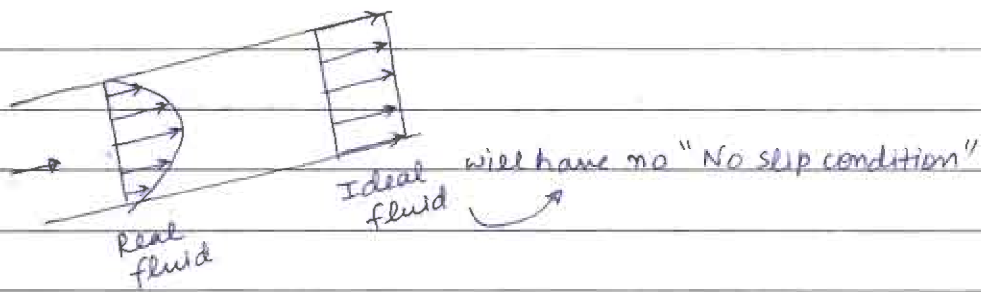
→ The continuum approach will not be applicable when the mean free path becomes large as compared to the characteristic dimensions of study. [alternate → Rarefied Gases Approach]

⇒ Ideal and Real Fluid

→ Fluid having no surface Tension, viscocity and which are incompressible are called ideal fluid.

→ There is no fluid as ideal fluid. It is just a theoretical conception.

⇒ No slip condition



→ A fluid having viscocity will be said to have no slip condition at the boundary of the solid surface i.e. the fluid molecule at the boundary will not move relative to the boundary surface.

→ Hence if boundary is stationary, fluid at the boundary will also be stationary and if boundary is moving, fluid at the boundary will also move with the same velocity in the same direction

NOTE: No wetting property is due to Surface Tension and not due to viscosity or no slip condition.

Properties of Fluid

⇒ Vapour pressure and cavitation

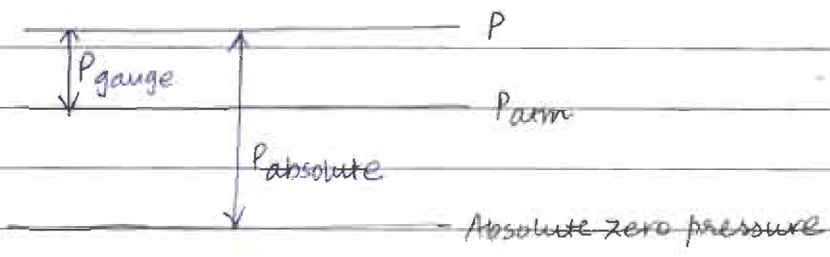
→ Saturation Vapour pressure is the pressure exerted by vapour molecules in phase equilibrium with it's liquid at a given temperature.

→ Saturation vapour pressure increases with increase in temperature and is independent of externally applied pressure.

→ whenever in a flow absolute pressure of flow exceeds the saturation vapour pressure the vapours and liquid will remain in dissolved form, however, if $P_{absolute} < P_{vap\ pressure}$, the dissolved gases and vapour will start coming out and will create cavity in the flow

→ These cavities move due to momentum of flowing fluid and when they go to high pressure zone, cavity collapses giving rise to noise, vibrations, surface pitting (erosion) and fatigue failure. This phenomenon is called cavitation.

→ It generally occurs at the inlet of pump, exit of reaction turbine, top of siphon and on the surface of spillway.



→ As the saturation vapour pressure increases with temperature, therefore chances of cavitation are more at higher temperature

→ Cavitation can be prevented by maintaining higher pressure in the flow and cavitation damage can be prevented by air entrainment.

→ Rise in Elevation, increase in velocity, decrease in atm pressure and increase in temperature will favour cavitation.

⇒ Bulk Modulus

$$\boxed{K = \frac{-dP}{\left(\frac{dV}{V}\right)}}$$

$$m = \rho V$$

$$dm = \rho dV + d\rho V = 0 \Rightarrow \boxed{\frac{-dV}{V} = \frac{d\rho}{\rho}}$$

$$K = \frac{dP}{\left(\frac{d\rho}{\rho}\right)} \Rightarrow \boxed{K = \rho \frac{dP}{d\rho}}$$

$$\boxed{\frac{1}{K} = \text{compressibility} = \frac{1}{\rho}\frac{d\rho}{dP}}$$

→ Ideal gas eq^n → $PV = nRT \Rightarrow \boxed{P = \rho RT}$

→ Isothermal condition

$$dP = RT\, d\rho \Rightarrow \boxed{\frac{dP}{d\rho} = RT}$$

$$\text{So,} \quad K = \rho \frac{dP}{d\rho} \Rightarrow K = \rho RT$$

$$\Rightarrow \boxed{K_{Isothermal} = P}$$

→ Adiabatic condition

$$P V^{\gamma} = \text{constant}$$

$$\frac{P}{\rho^{\gamma}} = \text{constant}$$

$$P = c\rho^{\gamma}$$

$$\frac{dP}{d\rho} = c\gamma\rho^{\gamma-1} \Rightarrow \rho\frac{dP}{d\rho} = c\gamma\rho^{\gamma} = \gamma P$$

So, $\boxed{K_{adiabatic} = \gamma P}$

→ adiabatic constant $= \frac{C_P}{C_V}$

C_P → specific capacity at constant pressure

C_V → specific capacity at const volume

Q- Density of sea water at free surface is 1030 kg/m^3 and atm. pressure is 98 KPa. Bulk modulus of elasticity of sea water is $2.34 \times 10^9 \; N/m^2$ (constant) and the variation of pressure with depth z is given by $\boxed{dP = \rho g dz}$ then determine the density and pressure at a depth of 2500 m.

$$K = \rho\frac{dP}{d\rho} \Rightarrow \boxed{dP = \frac{K d\rho}{\rho}} \Rightarrow \left[P\right]_{98000}^{P} = K \ln\left(\frac{1041.24}{1030}\right)$$

$P = \cancel{25487.21 \text{ KPa}}$

25495.21 KPa

So, $K\frac{d\rho}{\rho} = \rho g dz$

$$K\int_{\cancel{98000}\,1030}^{\rho}\frac{d\rho}{\rho^2} = \cancel{\rho}\, g\int_0^{2500} dz$$

$$K\left[\frac{-1}{\rho}\right]_{\cancel{98000}\,1030}^{\rho} = g(2500)$$

$$\frac{-1}{\rho} + \frac{1}{\cancel{98000}\,1030} = \frac{2500g}{2.34 \times 10^9}$$

P/F ⇑

$$\rho = \frac{1}{\frac{1}{\cancel{98000}\,1030} - \frac{2500g}{2.34 \times 10^9}} = 1041.24 \; kg/m^3$$

⇒ Viscosity

→ Viscosity is a measure of resistance of fluid to deformation. It is due to the internal frictional forces that develop b/w different layers of fluid that are forced to move relative to each other.

$$d\theta = \frac{du \cdot dt}{dy} \Rightarrow \boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

Rate of shear strain ($\frac{d\theta}{dt}$) ; Velocity gradient ($\frac{du}{dy}$).

→ Between the fluid layers, a shear stress develops due to resistance to movement. The fluid in which the shear stress developed is directly proportional to the rate of shear deformation is called Newtonian fluid

Hence, for Newtonian fluid ⇒ $\boxed{\tau \propto \frac{d\theta}{dt}}$

$$\Rightarrow \tau = \mu \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

→ μ is called the coefficient of viscosity or absolute viscosity or dynamic viscosity.

→ Unit of μ → $\frac{Ns}{m^2}$, $\frac{Kg}{m \cdot s}$, Pa·s, Poise $\boxed{10 \text{ Poise} = 1 \frac{Ns}{m^2}}$

NOTE: Dynamic viscosity of water is approx. 50 times that of air.

Kinematic viscosity ⇒ $\boxed{\nu = \frac{\mu}{\rho}}$

Unit → m^2/s , stoke [1 stoke = 1 cm^2/s]

NOTE: Kinematic viscosity of air is approx. 15 times that of water.

⇒ Effect of temperature on viscosity

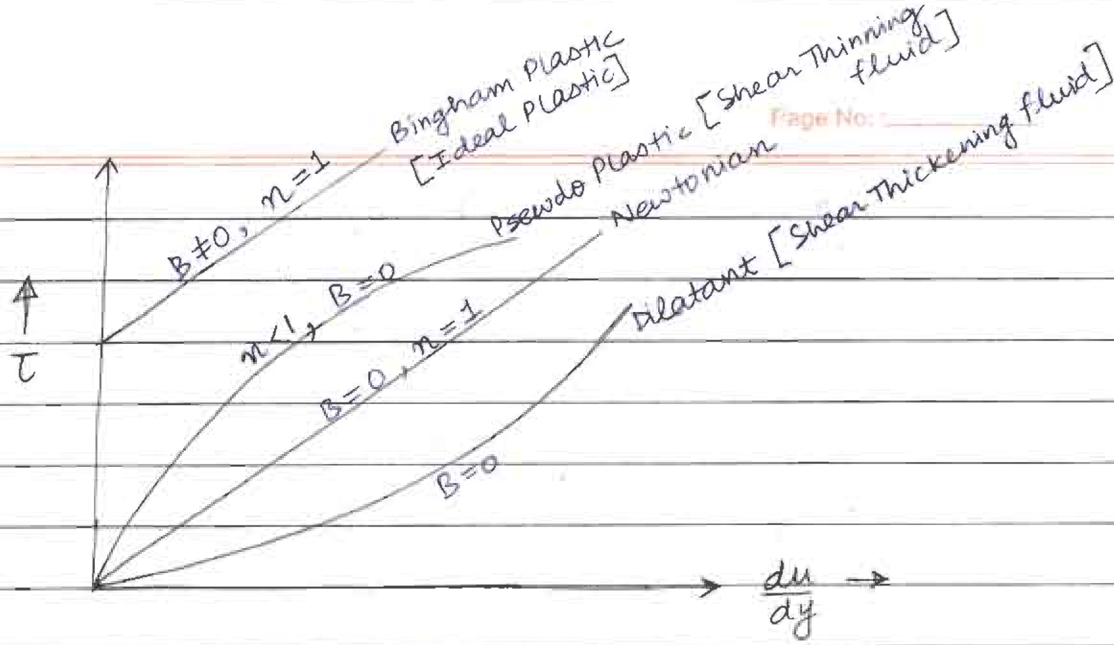
→ In case of liquids, cohesive force b/w the molecules cause viscosity. At higher temperature, molecules posses greater energy and hence can easily overcome the intermolecular cohesive forces, hence can move more freely, resistance to flow decreases.
Hence, for liquids, viscosity decreases with increase in temperature.

→ In case of gases, viscosity is caused by transfer of molecular momentum due to molecular collisions. Hence at higher temperature, since the no. of collisions per unit volume per unit time will be more, resistance to flow increases.
Hence, for gases, viscosity increases with increase in temperature.

→ For liquids, and gases, dynamic viscosity is independent of pressure.

⇒ Newtonian and Non-Newtonian fluid

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$



$$\frac{d(\tau)}{d\left(\frac{du}{dy}\right)} = An\left(\frac{du}{dy}\right)^{n-1} \Rightarrow \text{Apparent viscosity}$$

[Apparent viscosity = Dynamic viscosity for Newtonian fluid]

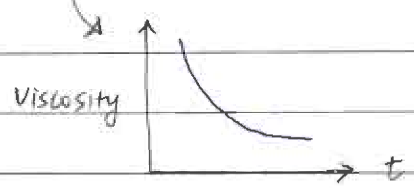
→ Newtonian → eg- water, air, gasoline, alcohol.

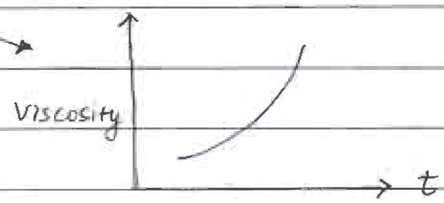
→ Dilatant → eg - solution with suspended starch or sand, sugar in water

→ Pseudo-Plastic → eg-Paints, polymer solution, blood, paper pulp, syrup, molasses, milk

→ Bingham Plastic → Bingham plastic fluid requires some minimum shear stress before they start moving called yielding.
Eg- Sewage sludge, toothpaste, drilling mud, mayonnaise.

→ Thixotropic and Rheopectic fluids have time dependent viscosity.





→ Printer's ink, certain paints and enamels, honey.

→ Gypsum paste, lubricants, synovial fluids.

→ Thixotropic fluids have time dependent pseudo plastic behaviour

→ Rheopectic fluids have time dependent dilatant behaviour.

→ Study of Non-newtonian fluids is called Rheology.

Q-

du/dy	0	0.5	1.1	1.8
τ	0	2	4	6

Identify the fluid.

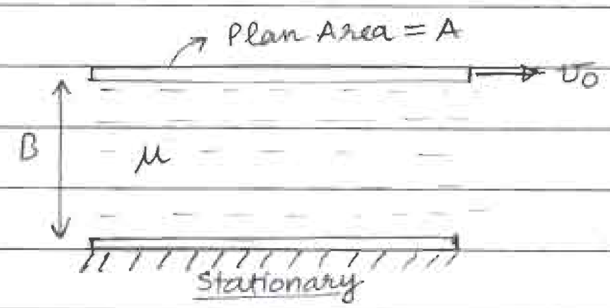
$d(du/dy)$	0.5	0.6	0.7
$d\tau$	2	2	2
Apparent viscosity	$\frac{2}{0.5}$	$\frac{2}{0.6}$	$\frac{2}{0.7}$

→ Apparent viscosity decreases as $\left(\frac{du}{dy}\right)$ increases.

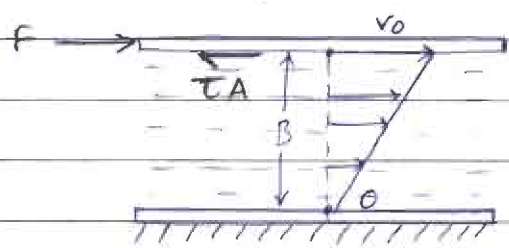
at the $\frac{du}{dy} = 0$, $\tau = 0$ +

There fore, Pseudo plastic fluid.
↳ shear thinning fluid.

Q-



Find the force required to maintain the motion of top plate.

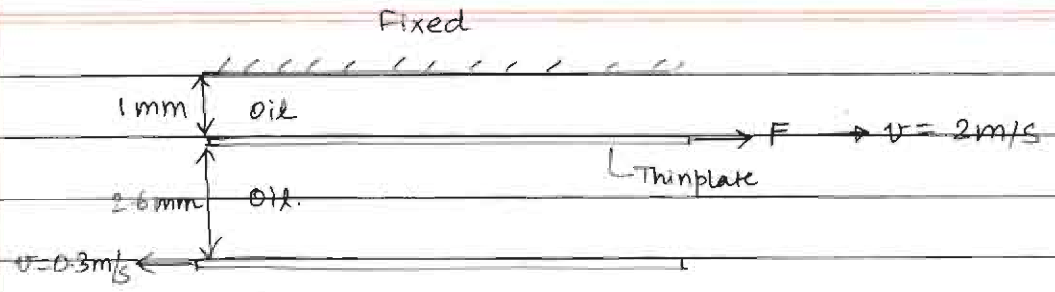


$$\frac{du}{dy} = \frac{v_0}{B}$$

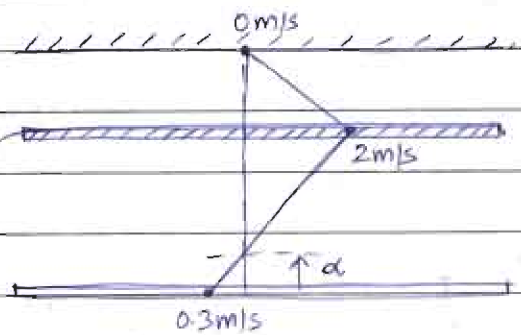
So, $\tau = \mu \frac{v_0}{B}$.

So, $\boxed{F = \tau A = \frac{\mu A v_0}{B}}$

Q-

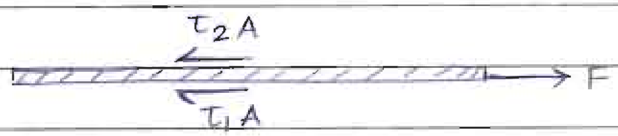


Thin plate has a plan Area of 40cm x 40cm
Assume the velocity in each oil layer to vary linearly.
Plot the velocity profiel and find the location where the velocity is zero. Also find the force that needs to be applied on the thin plate to maintain it's motion. Take $\mu = 0.027$ Pa.s.



$$\frac{0.3}{\alpha} = \frac{2}{2.6-\alpha} \Rightarrow 0.78 - 0.3\alpha = 2\alpha$$

$$\Rightarrow \alpha = \frac{0.78}{2.3} = 0.339 \text{ mm}$$



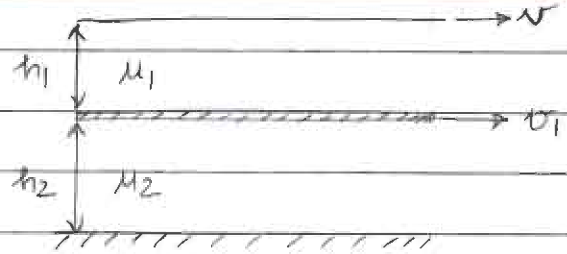
$$F = (\tau_1 + \tau_2) A$$

Now, $\tau_1 = \mu \left(\frac{du}{dy}\right)_1 = \mu \frac{2.3}{2.6 \times 10^{-3}} = \frac{23\mu}{26 \times 10^{-3}} = \frac{23000\mu}{26}$

$$\tau_2 = \mu \left(\frac{du}{dy}\right)_2 = \mu \frac{2}{1 \times 10^{-3}} = 2000\mu$$

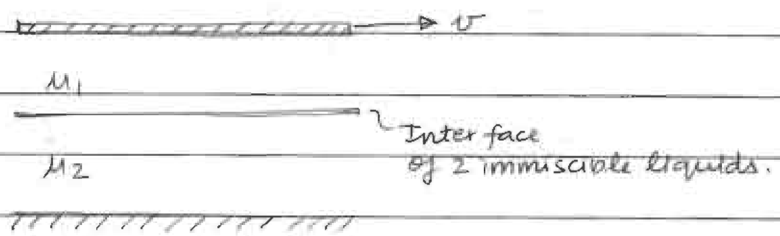
$$F = \left(2000\mu + \frac{23000\mu}{26}\right) \frac{40 \times 40}{10^4} = 12.46\text{N}$$

Q-



The top plate is being pulled with velocity v and the middle plate moves due to the pulling of upper plate with a uniform velocity v_1 $(v_1 < v)$. Find the ratio of v_1/v with the assumption that the velocity variation b/w the plates is linear

NOTE :



At the interface we will have → ① Continuity in velocity
② Continuity in shear.

Since, the middle plate is moving with constant velocity, net F = 0.

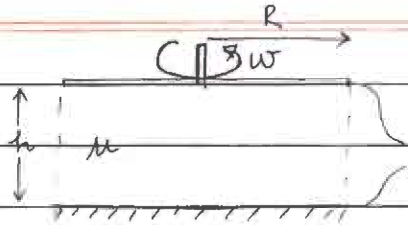
$$\mu_1\left(\frac{v-v_1}{h_1}\right)A$$

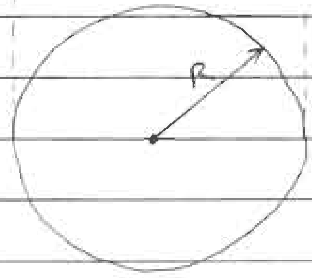
$$\left(\mu_2\,\frac{v_1}{h_2}\right)A$$

There fore, $\mu_1\left(\frac{v-v_1}{h_1}\right)\cancel{A} = \left(\mu_2\,\frac{v_1}{h_2}\right)\cancel{A}$

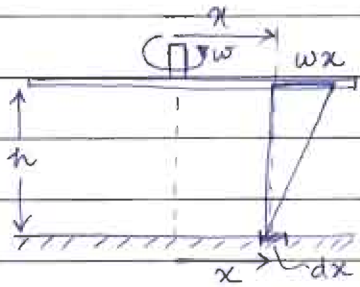
$$\boxed{\frac{v_1}{v} = \frac{\mu_1/h_1}{\mu_1/h_1+\mu_2/h_2} = \frac{1}{1+\frac{\mu_2}{\mu_1}\cdot\frac{h_1}{h_2}}}$$

Q-





Find the Torque experienced by the bottom plate.



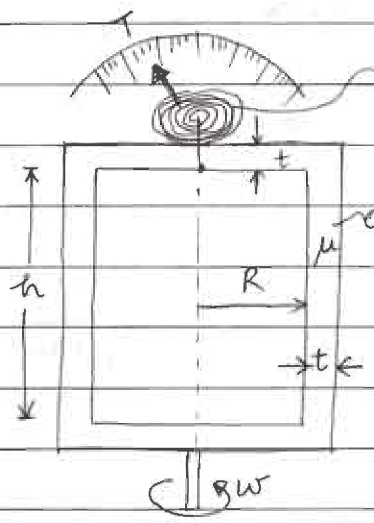
$$\left(\frac{du}{dy}\right)_x = \frac{\omega x}{h}$$

$$\tau = \mu \frac{\omega x}{h}$$

$$T = \int \tau \, dA \, x = \int \frac{\mu \omega x}{h} 2\pi x \, dx \cdot x$$

$$T = \frac{2\pi \mu \omega}{h} \int_0^R x^3 \, dx = \frac{\pi \mu \omega R^4}{2h}$$

NOTE: Rotating cylinder viscometer



$$T_{bottom} = T_{top} = \frac{\pi \mu \omega R^4}{2t}$$

For calculation of T_{side}

$$\tau = \mu \frac{\omega(R+t)}{t} \quad , \quad F = \frac{\mu\omega(R+t)}{t}(2\pi R h)$$

$$T_{side} = \frac{\mu\omega(R+t)(2\pi R h)}{t} \times R$$

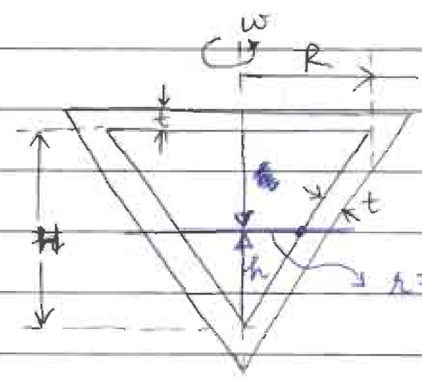
So, Torque experienced by the inner cylinder,

$$\boxed{T = 2\left[\frac{\mu\pi\omega R^4}{2t}\right] + \frac{\mu\omega(R+t)(2\pi R h)}{t} \cdot R}$$

measured from calibrated rotational spring

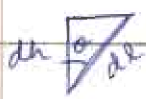
$\Rightarrow$ Hence μ of the fluid can be calculated.

$\Rightarrow$

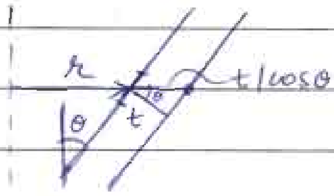


$r = \frac{Rh}{H}$

$$T_{top} = \frac{\pi \mu \omega R^4}{2t}$$



$dl = \frac{dh}{\cos\theta}$



$$\frac{du}{dy} = \frac{\omega\left(r + \frac{t}{\cos\theta}\right)}{\left(\frac{t}{\cos\theta}\right)}$$

$$\tau = \frac{\mu\omega\left(r + \frac{t}{\cos\theta}\right)}{\left(\frac{t}{\cos\theta}\right)}$$

$$\boxed{T_{side} = \int_0^H \frac{\mu\omega\left(r + \frac{t}{\cos\theta}\right)}{\left(\frac{t}{\cos\theta}\right)} \, 2\pi r \left(\frac{dh}{\cos\theta}\right) r}$$

$\rightarrow$ Put $r = \frac{Rh}{H}$